Hence the total capacitance C_e of the system in the s plane in terms of the radii of the small semicircles in the s plane is

$$C_e = \frac{2}{\Pi}\{\log\,[2(l^2 - 1)/l] - \log \delta s\} \tag{7}$$

since $\delta r = 2s\delta s$.

By definition, the even-mode fringing capacitance, C_{f_e}'', is given by the limiting value of $C_e - C_{P_A} - C_{P_D}$, where C_{P_A} and C_{P_D} are the parallel-plate capacitances associated with the gaps B_1 and B_2 of Fig. 1. In [1], C_{f_o}'' was defined as $C_o - C_{P_A} - C_{P_D}$ and, since the formulas for the parallel-plate capacitances are rather involved, it is convenient to express C_{f_e}'' in terms of C_{f_o}''. Thus

$$C_{f_e}'' = C_{f_o}'' + C_e - C_o. \tag{8}$$

From [1, eq. (12)]

$$C_o = \frac{1}{\Pi}\{2\log(\nu - \mu) - \log \delta\mu - \log \delta\nu\}. \tag{9}$$

Now $\delta\mu$ and $\delta\nu$ are obtained from δs with the help of (1). After differentiating, it is found that

$$\delta\mu = \frac{(\mu - \beta)^2}{\gamma(\alpha - \beta)}\,\delta s \tag{10}$$

and

$$\delta\nu = \frac{(\nu - \beta)^2}{\gamma(\alpha - \beta)}\,\delta s. \tag{11}$$

Then from (7) and (9)

$$C_o - C_e = \frac{2}{\Pi}\left\{\log(1 - k^2\,\mathrm{sn}^2\,a\,\mathrm{sn}^2\,d) - 2\log(k\,\mathrm{sn}\,d) - \log\frac{2(l^2-1)}{l} - \log\frac{(\mu - \beta)(\nu - \beta)}{\gamma(\alpha - \beta)}\right\}. \tag{12}$$

This expression, together with [1, eq. (13)], in view of (8) gives the desired formula for C_{f_e}''.

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An Analytical Comparison of Two Simple High-Q Gunn Oscillators

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Abstract—**This note compares and analyzes two commonly used simple waveguide Gunn oscillators in terms of their loaded Q-factors. Suitable design criteria are established for both, and two oscillators which were tested conformed well to these. It is concluded that although the more mechanically complex oscillator, which is in common use, has a greater flexibility, the simpler oscillator is adequate for most applications.**

I. Introduction

The Gunn diode is a simple two-terminal device which, when mounted in a resonant circuit and biased with a suitable dc potential, generates microwave power. The basic noise and stability properties of the device are modified by the loaded Q-factor (Q_L) of the resonant circuit and for many applications a desirable value of Q_L is between 200 and 1000. Resonant circuits, or cavities, for this purpose are usually made from simple waveguide and Figs. 1 and 2 show two common types. The purpose of this short paper is to analyze the critical design aspects of these cavities and to determine if either has any basic advantages.

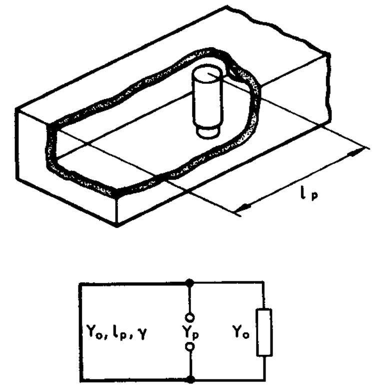

Fig. 1. Post-type oscillator and its equivalent circuit.

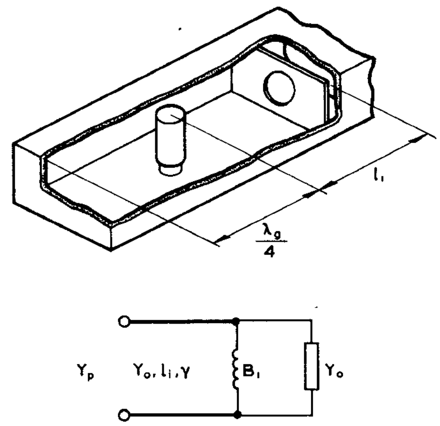

Fig. 2. Iris-coupled oscillator.

The oscillator shown in Fig. 1 has been previously studied [1] and there are many commercial samples of this type. It consists simply of a post-mounted Gunn diode spaced a half wavelength from a short circuit.

The second oscillator, which is mechanically more complex, consists of a Gunn post assembly mounted between a simple inductive (i.e., circular hole) iris and a waveguide short circuit, Fig. 2. There are also many oscillators of this design commercially available and it is commonly supposed [2] to have advantages over the more simple post-coupled oscillator.

Although the two oscillators appear simple in construction, the analyses are complex. A numerical analysis of the post-mounting structure was given by Eisenhart and Kahn [3] in 1971, and this analysis is used for final evaluation of both oscillators. However, a more basic analytical approach is adopted in this short paper in order to give a meaningful comparison of the two cavities. The simplified analysis is only concerned with the circuit external to the post since complex effects of the post are the same for both circuits. In this respect it differs from the analysis of White [4] and leads to simple expressions for the oscillators' Q-factors. The interface reference plane is at the waveguide/post junction Y_p representing the admittance seen by the "Gunn–package–post." The equivalent

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circuits of Figs. 1 and 2 are therefore only for the circuit external to the complex post.

The Q-factor of the simple post-coupled oscillator is initially related to the waveguide short-circuit position, while that of the iris-coupled oscillator is related to the normalized iris susceptance. It is then shown that the loaded Q-factor of each can be related to the reflection coefficient of the Gunn post assembly and that both oscillators degenerate to the same case.

The results of the analysis show that for most applications the mechanically simpler mount is equal in performance to the iris-coupled type. However, the iris-coupled type is more adaptable, and the use of the simple post-coupled type is restricted to the Gunn device because the Gunn diode is capable of generating near-optimum output power into a wide range of impedances.

II. Analysis of the Post-Mounted Cavity

For the purpose of this analysis the equivalent circuit depicted in the inset of Fig. 1 is used. The Gunn device, together with the post and package is represented by a parallel combination of a susceptance and a negative conductance shunting the waveguide. The short-circuited lossy line, representing the waveguide, presents an admittance $G_{S/C} + jB$ at the post terminals, where

$$G_{S/C} = \frac{Y_0 \sinh 2\alpha l_p}{\cosh 2\alpha l_p - \cos 2\beta l_p} \tag{1a}$$

$$B = \frac{Y_0 \sin 2\beta l_p}{\cosh 2\alpha l_p - \cos 2\beta l_p} \tag{1b}$$

and

Y_0 ≡ the waveguide characteristic admittance;
α ≡ loss/unit length of the waveguide;
l_p ≡ distance between the short circuit and the post;
β ≡ the waveguide phase constant.

The susceptance $B(\omega)$ of the short circuit, the conductance $G_{S/C}$, and the load conductance Y_0, are connected in parallel across the post, and the load admittance terminating the post is $G_{S/C} + Y_0 + jB(\omega)$. Hence we can calculate the loaded Q-factor by the relation[1]

$$Q_L = \frac{\omega}{2G}\frac{\partial B(\omega)}{\partial \omega} = \frac{\omega}{2(G_{S/C} + Y_0)}\frac{\partial B(\omega)}{\partial \omega}.$$

But the waveguide unloaded Q-factor, Q_u, can also be defined from $\partial B(\omega)/\partial\omega$, as

$$Q_u = \frac{\omega}{2G_{S/C}}\frac{\partial B(\omega)}{\partial \omega}.$$

Hence

$$Q_L = \frac{Q_u G_{S/C}}{(G_{S/C} + Y_0)} \tag{2}$$

and substituting (1) into (2)

$$Q_L = \frac{Q_u \sinh 2\alpha l_p}{[\exp(2\alpha l_p) - \cos 2\beta l_p]}. \tag{3a}$$

Ignoring dispersion effects in the waveguide, i.e., $\lambda_g = \lambda_0$, then as $\alpha \to 0$

$$Q_L = \frac{Q_u \alpha l_p}{\sin^2 \beta l} = \frac{\omega l_p}{2c \sin^2\left(\frac{2\pi l_p}{\lambda_0}\right)} \tag{3b}$$

[1] Arguments continue concerning the inclusion of an extra factor of 2 in this expression [5]. However, to remain consistent with the expressions used for measuring loaded Q-factor given by Warner and Hobson it has been omitted here.

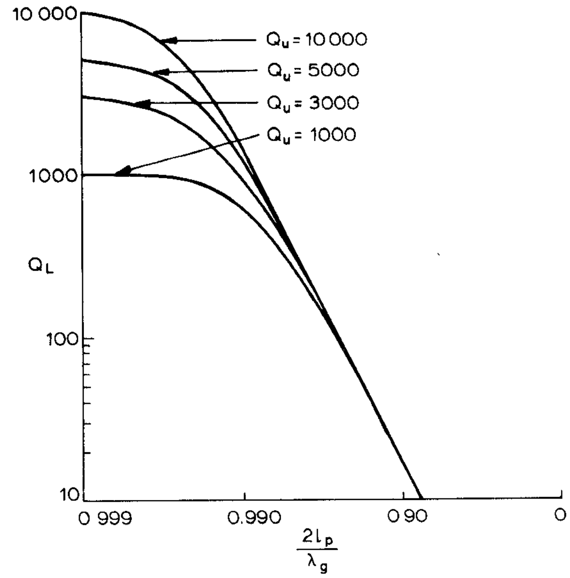

Fig. 3. Loaded Q-factor Q_L of the post-coupled oscillator as a function of short-circuit position, normalized to half the operating wavelength, λg, with the waveguide unloaded Q-factor Q_u as a parameter.

since

$$Q_u = \frac{\pi}{\alpha\lambda_g} = \frac{\omega}{2\alpha c}.$$

The dependence of Q_L upon cavity length as defined by (3) is illustrated in Fig. 3. It is significant that, for high Q operation, the distance l_p between short circuit and post should be close to $\lambda g/2$.

III. Analysis of the Iris-Coupled Oscillator

For the analysis of this oscillator the equivalent circuit depicted in the inset of Fig. 2 is used. In the oscillator considered, the short circuit is positioned a quarter wavelength from the post and presents an open circuit in the plane of the post and does not therefore enter the high Q-analysis. The inductive iris positioned between the post and the load can be represented by a shunt negative susceptance, $-B_i$ [6], and it appears in parallel with the load conductance Y_0. Hence there is a complex admittance $Y_0 - jB_i = Y_L$ a distance l_i from the Gunn post. The admittance presented in the plane of the Gunn post, Y_p, by the admittance Y_L is given by

$$Y_p = Y_0 \frac{Y_L + Y_0 \tanh \gamma l_i}{Y_0 + Y_L \tanh \gamma l_i} = G + jB. \tag{4}$$

where γ is the complex propagation constant of the waveguide. It is now possible to calculate the loaded Q-factor from the expression $Q_L = (\omega/2G)/(\partial B/\partial\omega)$ as

$$Q_L = \frac{Q_u \alpha l_i}{[\alpha l_i + (Y_0/B_i)^2]} \tag{5}$$

which simplifies to

$$Q_L = \frac{\omega l_i}{c(Y_0/B_i)^2} \tag{6}$$

if we assume no dispersion and also let $\alpha \to 0$. Fig. 4 is a graph of the oscillator-loaded Q-factor, against the inverse of the normalized iris admittance squared, $(Y_0/B_i)^2$, for various waveguide unloaded Q-factors (Q_u). It is assumed that l_i is approximately one-half wavelength long. Fig. 4 indicates that (Y_0/B_i) must be small to achieve a high loaded Q-factor. This criterion

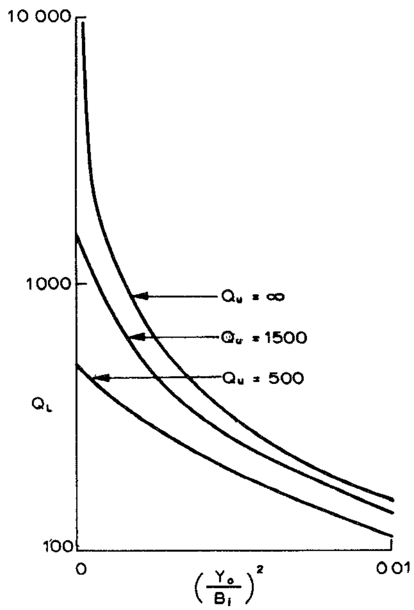

Fig. 4. Plot of the iris-coupled oscillator-loaded Q-factor Q_L against the square of the inverse normalized iris susceptance, $(Y_0/B_i)^2$ with the cavity unloaded Q-factor Q_u as a parameter.

is equivalent to the requirement that the short-circuit separation from the Gunn post must approach a half wavelength in the post-coupled oscillator.

IV. Analyses of Both Oscillators Using Reflection Coefficients

It is obvious that the inductive iris with susceptance $-B_i$ is not a unique means of providing a resonant cavity for the Gunn post. Alternative techniques could use other iris shapes or perhaps a metal screw or waveguide-admittance change, providing that the discontinuity–post distance is appropriately altered. Such methods provide decoupling of the Gunn post from the load by reactive mismatching, and the resulting admittance presented to the post can be characterized by a reflection coefficient ρ. The admittance presented to the post in the post-coupled oscillator can also be represented by a complex reflection coefficient, ρ_p, and oscillator characterization in terms of reflection coefficients is therefore a convenient and suitable means of comparing the two oscillator configurations.

For the simple post-coupled oscillator the admittance presented to the post terminals is

$$Y_p = Y_0(1 - j \cot \beta l_p)$$

which is comprised of a lossless waveguide with a short-circuit termination in parallel with the waveguide load. Therefore

$$\rho_p = \frac{(Y_0 - Y_p)}{(Y_0 + Y_p)} \tag{7}$$

and

$$\sin^2 \beta l_p = \frac{(1 - |\rho_p|^2)}{(1 + 3|\rho_p|^2)}. \tag{8}$$

Thus from 3(b)

$$Q_L = \frac{\omega l_p(1 + 3|\rho|^2)}{2c(1 - |\rho_p|^2)}. \tag{9a}$$

For the iris-coupled oscillator we have a discontinuity due to the iris a distance l_i from the Gunn post. Defining this as a reflection coefficient ρ_i and relating it to the oscillator Q-factor, the magnitude of the reflection coefficient transferred from the iris to the post is given by

$$|\rho_p| = |\rho_p| \exp(-2\alpha l_i).$$

An arbitrary phase angle ϕ is assumed for the reflection coefficient at the post which, when expanded as a normalized impedance, includes two series-real terms. These terms are related to the cavity loss and the waveguide load. The power distribution between cavity loss (P_{abs}) and load (P_L) is given by the expression

$$\frac{P_{\text{abs}}}{P_{\text{avail}}} = \frac{4\alpha l_i|\rho_i|^2}{1 - |\rho_i|^2 + 4\alpha l|\rho_i|^2}$$

where P_{avail} = total power generated = $P_L + P_{\text{abs}}$. Now the oscillator-loaded Q-factor may be defined [8] as

$$Q_L = \frac{Q_u P_{\text{abs}}}{P_{\text{avail}}}$$

and assuming no dispersion, then

$$\lim_{\alpha \to 0} | Q_L = \frac{\omega l_i 2|\rho_i|^2}{c(1 - |\rho_i|^2)}. \tag{9b}$$

For high Q operation it is necessary for the magnitude of the reflection coefficient to approach unity, and (9a) and (9b) simplify to

$$Q_L = \frac{2\omega l}{c(1 - |\rho_p|^2)} \tag{9c}$$

since $|\rho_i| = |\rho_p|$, when $\alpha = 0$.

Hence we have identical Q-factors for the oscillators if $l_p = l_i$. For the post-coupled oscillator $l_P \to n\lambda g/2$ as $|\rho_p| \to 1$. In practice, in order to avoid multiple resonances, the $n = 1$, $\lambda g/2$ solution for l_p is used. For the iris-coupled oscillator, it can be shown that because the iris is a shunt-mounted element, as $|\rho_i| \to 1$ then $\underline{/\rho_i} \to 180°$. It then follows that similar admittances are presented to the post terminals in both the iris-coupled and the post-coupled cases if $l_i \to n\lambda g/2$. In practice, therefore, $l_p \simeq l_i \simeq \lambda g/2$ and (9c) becomes, for both oscillators

$$Q_L = \frac{2\pi}{(1 - |\rho_p|^2)}. \tag{9d}$$

This equation relates the loaded Q-factor Q_L to the admittance presented at the post terminals in terms of the modulus of reflection coefficient only.

At the post terminals the load is defined as $Y_p = G + jB$ which equals $-Y_G$, the admittance presented by the Gunn. Therefore, to oscillate with a given Q-factor both the load and the Gunn device must present appropriate admittances at the post terminals, and the ability of the Gunn to provide this is the prime limitation to the loaded Q-factor. Once this admittance is defined, circuits which provide an appropriate load will automatically have the required Q-factor. The next section concerns the admittance ranges available with the two simple oscillators.

V. Practical Oscillators

The foregoing analyses assumed a constant negative conductance and susceptance representation of the Gunn diode and post at the post terminals. In practice, this is not strictly true but the assumption is valid providing that the Q-factor of the Gunn post Q_p is smaller than Q_L. The calculations of the Gunn post impedance in Table I confirm that this condition is satisfied. However, the post and device packaging influence the oscillator behavior in the following two ways.

TABLE I
CALCULATED POST-TERMINAL ADMITTANCES, LOADED Q-FACTORS, AND POST Q-FACTORS FOR VARIOUS POST DIAMETERS IN WG-18 AT 11.6 GHz

Post diameter (mm)	Diode chip negative resistance Rg (Ω)	Post terminal admittance $G/Y_o + jB/Y_o$	Post Q-factor $Q_p\left(=\frac{\omega}{2G}\frac{\partial B}{\partial \omega}\right)$	Post Q-factor Q_L (from eqn. 9c)
6	-100	-20.86 31.13	0.25	63
6	-300	-11.30 36.06	4.1	196
6	-1000	-3.65 39.04	16	652
5	-100	-27.32 22.37	0.5	68
5	-300	-16.79 44.74	3.8	210
5	-1000	-5.57 49.94	17	712
4	-100	-45.29 11.08	0.2	72
4	-300	-70.13 71.24	0.02	221
4	-1000	-35.93 125.9	15	746
3	-100	-23.41 -24.70	0.1	75
3	-300	-16.93 -46.77	4	226
3	-1000	-5.86 -53.05	17	764
2	-100	-3.38 -12.53	1.4	73
2	-300	-1.32 -13.86	5.2	227
2	-1000	-0.41 -14.04	17	766

Firstly, there is a phase change between the negative resistance, or active part of the Gunn diode and the post terminals, which means that in practice measured post to obstacle distances differ slightly from the distances calculated using (4) to (9).

Secondly, modification of the post dimensions can alter the negative terminal conductance at the Gunn post and thus enable the Gunn device to work into a range of admittances.

An additional and larger variation of terminal impedance is also available because the Gunn diode is capable of providing near-optimum output power into a large range of working impedances. For example, a CXY19 commercial Gunn diode measured by Gough [7] gave 184 mW of power with a negative conductance of 3 mS and 220 mW with $G_D = -10$ mS. Copeland [9] predicted similar variations in a computer simulation. This property of variable negative conductance for only a small loss of output power enables the Gunn post-terminal admittance to cover a large range of admittances. This is shown clearly in Table I. The post-terminal admittances at 11.6 GHz in Table I were derived using the analysis of Eisenhart and Kahn [3] and packaged Gunn-diode measurements of Gough [7]. Only the post diameter was varied because of practical considerations of heat sinking the diode to the waveguide wall and avoiding oscillator harmonics exciting higher order waveguide modes.

Conjugate admittances of Table I are plotted on the Smith chart of Fig. 5 with superimposed constant Q-factor circles. It can be seen that a large range of admittances can be catered for by varying the Gunn-diode post diameter or the Gunn-diode negative resistance. Thus the post-coupled oscillator which has an admittance locus $Y_0(1 + jB)$ can provide oscillators with $Q_L = 315$ for a 2-mm post to $Q_L = 652$ for a 2.5-mm post while maintaining a near-optimum output power, since the

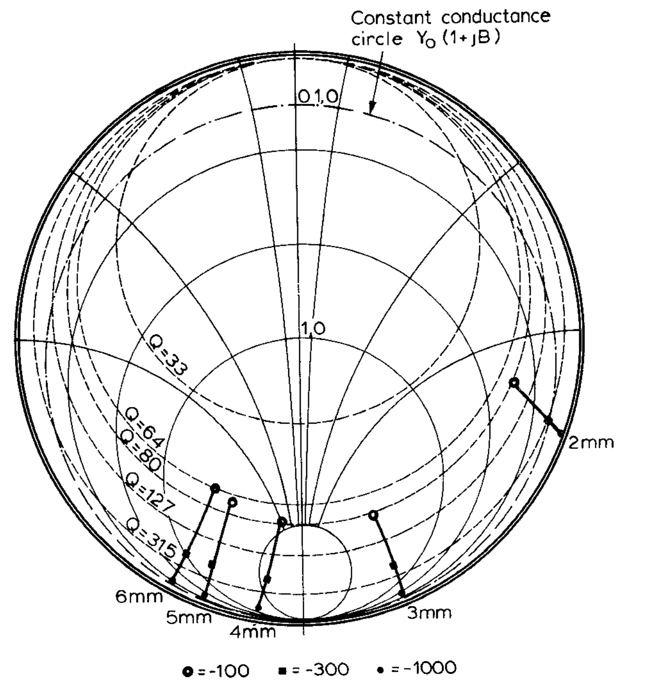

Fig. 5. Conjugate post-terminal impedances for various post diameters with R_g, the Gunn-diode resistance as a parameter, and the constant Q-factor circles on a Smith chart normalized to $10Y_0$.

negative-diode chip resistance only varies from -300 to -1000 Ω. An experimental oscillator built with a 2.2-mm post diameter had a Q-factor of 500 and 100-mW output power at 11.6 GHz with a CXY19 Gunn diode. These data agreed well with the predicted data from the analysis.

An iris-coupled oscillator can match any post-terminal admittance, and a cavity was constructed to oscillate at 11.6 GHz with a Q-factor of 625 using a 4-mm diameter post. An output power of 90 mW at 11.5 GHz was obtained experimentally with a Q-factor of 600 with an iris susceptance of $-20Y_0$ located 0.46 λg from the Gunn post terminals. This result confirmed the design procedure but the iris-to-post distance was slightly in error since post thickness effects were ignored [6]. The design procedure also indicated that the Gunn-diode negative susceptance required, if a 4-mm post was used in a post-coupled cavity, was too low (≪ 1 mS) to provide usable output power and this has also been experimentally confirmed.

VI. CONCLUSIONS

The design and analysis of two simple waveguide oscillators has been described. The two oscillators are shown to be equivalent but the iris-coupled type has shown greater flexibility in design. However, in practice it has been seen that the flexibility of the Gunn device in providing near-optimum output power over large working impedance ranges enables the simpler post-coupled oscillator to produce a similar performance to that of the iris-coupled oscillator. The Q-factor can be varied in the post-coupled oscillator by adjustment of the post diameter and short-circuit position. In the iris-coupled oscillator it is achieved by varying the iris hole size and compensating with cavity length or post diameter changes.

Since the Q-factor, and hence the "performance," of the oscillator is related to the reflection coefficient at the post terminals, the performance is independent of the cavity used providing the post admittance can be matched. It is therefore concluded that while the iris-coupled oscillator has greater flexibility, the cheaper construction of the post-coupled oscillator is often adequate.

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Radiation Losses of Planar Circuit Resonators and the R/Q Parameter

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***Abstract*—The resonator parameter R/Q, the ratio of a shunt resistance to the unloaded Q, which might be termed a "mode-geometry" parameter, is a natural parameter for characterizing oscillation modes of planar-circuit resonators which are open circuited at the edges. These resonators are often excited by connection at the edge to a microstrip transmission line, and the appropriate shunt resistance is the equivalent resistance at resonance at these terminals for that mode.**

Losses in planar-circuit resonators include the ohmic (skin-effect and dielectric) losses of enclosed resonators plus a radiation-loss component. For a variety of planar resonators, the ohmic losses are easily calculated, but the radiation-loss determination is a difficult boundary-value problem. More specifically, the determination of either the unloaded Q or the radiation component of the unloaded Q is often readily accessible only through measurement. The knowledge of the R/Q parameter allows one, in effect, to replace the Q measurement with a shunt-resistance measurement, which is often more expedient to perform.

Two simple planar-resonator configurations, the circular disk, and the square plate are studied. The radiation component of the Q is evaluated by using the measured shunt resistance and the R/Q parameter to calculate the total unloaded Q and thence the radiation component of the unloaded Q. A comparison is made between these results and those obtained from direct Q measurements.

INTRODUCTION

The direct calculation of radiation loss from planar circuits is an extremely difficult boundary-value problem. Radiation from microstrip can be treated by regarding the strip as a line source of current [1] and an extension of this approach has been used [2]; however, even the simplest planar resonator, the circular disk, is not amenable to this approach. Alternative routes toward a solution of the problem of losses in open resonators will be useful.

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THE GEOMETRICAL CAVITY FACTOR R/Q

Resonators having a Q, which is sufficiently large so that the configuration of the electromagnetic field is substantially the same as in the absence of losses, can be characterized by a ratio of a shunt resistance to a Q which is essentially geometrical, i.e., it is independent of cavity losses [3]. In the case of an open-circuited planar resonator it is convenient to define the shunt resistance in terms of the voltage at the point at which it will be excited by connection to a microstrip line, typically at the edge of the resonator. Thus, if the total time-average power dissipation due to conductor losses, dielectric losses, and radiation losses is P and the peak amplitude of the voltage at the excitation point is V, the shunt resistance is

$$R = \frac{V^2}{2P}. \tag{1}$$

The unloaded Q is

$$Q = \omega_0 \frac{U}{P} \tag{2}$$

where U is the energy stored in the cavity and P is the previously mentioned power dissipation at resonance; whence

$$R/Q = \frac{V^2}{2\omega_0 U}. \tag{3}$$

This is a result which is dependent on the mode type but independent of losses as long as they do not cause significant modifications to the spatial distribution of the fields within the cavity. It is valid for any planar configuration, whether it is open circuited or short circuited at the edges of the plane, to the extent that edge effects represent a small perturbation to the otherwise known field distribution.

For a cavity for which the shunt resistance can be measured and the right-hand side of (3) can be calculated, the Q can then be found and, if the Q for conductor loss and dielectric loss can be determined by other means, the radiation loss can be found from the fact that

$$\frac{1}{Q} = \frac{1}{Q_{\text{cond. loss}}} + \frac{1}{Q_{\text{diel. loss}}} + \frac{1}{Q_{\text{rad. loss}}}. \tag{4}$$

Examples

The preceding approach can be readily used with open-circuited planar resonators of simple geometrical configuration.

1) Circular Disk: One such resonator is the circular disk which is illustrated in a cross-sectional view in Fig. 1. In the dominant mode, it contains an electric field lying in the z direction only and given by

$$E_z = E_0 \cos \phi J_1(kr) \tag{5}$$

where r and ϕ are the usual circular cylindrical coordinates. The open-circuit edge condition requires that $J_1'(ka) = 0$ or $ka = 1.841$, hence the edge voltage $V = E_0 b J_1(1.841)$. There are two components of the magnetic field, H_ϕ and H_r, which are not needed for (3) but are needed to separate the losses into their components. The energy stored in this mode is obtained by integrating the energy density $\frac{1}{4}\varepsilon|E|^2$ over the cavity volume and gives the result

$$U = E_0^2 \frac{\pi}{4} \varepsilon b a^2 \left(1 - \frac{1}{(ka)^2}\right) J_1^2(ka) \tag{6}$$

whence

$$\frac{R}{Q} = \frac{2}{\pi} \eta \frac{b}{a} \frac{ka}{(ka)^2 - 1} \tag{7}$$